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Dimension-2 condensates and Polyakov Chiral Quark Models

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Abstract. We address a possible relation between the expectation value of the Polyakov loop in pure gluodynamics and full QCD based on Polyakov Chiral Quark Models where constituent quarks and the Polyakov loop are coupled in a minimal way. To this end, we use a center symmetry-breaking Gaussian model for the Polyakov loop distribution which accurately reproduces gluodynamics data above the phase transition in terms of dimension-2 gluon condensate. The role played by the quantum and local nature of the Polyakov loop is emphasized.

PACS. 12.39.Fe Chiral Lagrangians – 11.10.Wx Finite-temperature field theory – 12.38.Lg Other non-perturbative calculations

The phase transition of QCD matter at finite temperature from hadrons to a quark-gluon plasma was established long ago [1–3] (for a review see, *e.g.*, [4]). The precise definition of the phase transition requires a proper identification of the relevant order parameters. In the heavy-quark limit, the Polyakov loop vacuum expectation value signals the breaking of the center symmetry corresponding to the deconfinement phase transition when changing from zero to one [5]. In the limit of light quarks the chiral condensate determines the restoration of chiral symmetry when the chiral condensate vanishes above the critical temperature. In the real QCD case with dynamical massive quarks both chiral and center symmetries are explicitly broken, and neither the Polyakov loop nor the chiral condensate are truly order parameters, although a rather sharp crossover is expected across the phase transition for these quantities [4].

Polyakov-Chiral Quark Models allow to study the interplay between chiral symmetry restoration and center symmetry breaking [6] in a quantitative manner [7–13]. At zero temperature the constituent quark mass, M, is dynamically generated via the spontaneous breaking of chiral symmetry, inducing a exponentially small, $\sim e^{-M/T}$, breaking of the center symmetry at low temperatures [9–11]. This provides the rationale for keeping the Polyakov loop as an order parameter also in the unquenched case. However, although the coupling of the Polyakov loop to quarks is rather unique, details regarding the postulated purely gluonic action differ [7–13]. This additional information is beyond the chiral quark model capabilities and must always be postulated. In this regard, it seems natural to constrain the gluonic action to reproduce pure gluodynamics results. In the present work we address a possible relation between the expectation value of the Polyakov loop in pure gluodynamics and full QCD based on the coupling to quarks generated by the fermion determinant within a Polyakov NJL model (PNJL).

One of the problems one must also face when comparing models with lattice data has to do with the difficult but necessary renormalization of the Polyakov loop. Indeed, after renormalization the Polyakov loop falls outside the unitary group [14,15]. The spontaneous breaking of the center symmetry above some critical temperature occurs already at the level of pure gluodynamics and is interpreted as the signal of deconfinement [16]. Full dynamical QCD lattice simulations account, in addition, for an explicit center symmetry breaking at low temperatures due to the presence of fermions [17]. In a recent paper [18] we have shown that the behaviour of the Polyakov loop above the deconfinement phase transition can be naturally accommodated by a dimension-2 condensate, instead of the more conventional perturbative calculations. The quality of our fits leaves little doubt on the veracity of the description of the lattice data. To our knowledge this is the first time that power corrections in temperature have been advocated to describe QCD right above the deconfinement phase transition. This is in sharp contrast to the common believe that logarithmic thermal corrections should be considered instead yielding to rather unnatural, in fact inaccurate, fits to the existing lattice results.

Following our study on the role of dimension-2 condensate in the center symmetry-breaking as a guideline [18] we make here a Gaussian ansatz for the purely gluonic contribution to the Polyakov loop distribution which reproduces remarkably well the lattice data for pure glu-

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odynamics [16]. Then, using the PNJL model [7–13] we may *predict* the value of the Polyakov loop in the *unquenched* case and compare with full dynamical calculations. In our view this is the best possible scenario for a Polyakov-Chiral quark model to work.

Let us explain how the dimension-2 gluon condensate can generate a powerlike behaviour of the Polyakov loop at temperatures slightly above the deconfinement phase transition in gluodynamics. In the Polyakov gauge the (expectation value of the) Polyakov loop is defined as

$$L = \left\langle \frac{1}{N_c} \mathrm{tr}_c \, e^{igA_0(\boldsymbol{x})/T} \right\rangle , \qquad (1)$$

where $\langle \ \rangle$ denotes vacuum expectation value and tr_c is the (fundamental) color trace. A_0 is the gluon field in the (Euclidean) time direction, $A_0 = \sum T_a A_{0,a}$, T_a being the Hermitian generators of the $SU(N_c)$ Lie algebra in the fundamental representation, with the standard normalization $\mathrm{tr}_c(T_a T_b) = \delta_{ab}/2$. Note that in this gauge we have the invariance under the large gauge transformation $gA_0 \to gA_0 + 2\pi T$.

If we expand the Polyakov loop at high temperatures we get the cumulant expansion

$$L = \sum_{k=0}^{\infty} \frac{1}{(2k)!} \left(\frac{-g^2}{T^2} \right)^k \langle A_{0,a_1} \dots A_{0,a_{2k}} \rangle \times \frac{1}{N_c} \mathrm{tr}_c (T_{a_1} \dots T_{a_{2k}}), \qquad (2)$$

where odd terms have been discarded on the assumption that global colour symmetry breaking does not occur. Averaging over the $N_c^2 - 1$ colour gluonic degrees of freedom we get

$$\langle A_{0,a_1} \dots A_{0,a_{2k}} \rangle = \frac{\langle (A_{0,a})^{2k} \rangle (N_c^2-3)!!}{(N_c^2+2k-3)!!} \delta_{a_1,\dots,a_{2k}} \qquad (3)$$

where $\delta_{a_1,\dots,a_{2k}}$ is the fully symmetrized product of the Kronecker delta symbols δ_{ab}. The $SU(N_c)$ Casimir invariant is $C_2 = \sum_a T_a T_a = (N_c^2-1)/(2N_c)\mathbf{1}_{N_c}$. In the large N_c limit only the contractions of adjacent indices (including cyclic permutations) survive. For instance, if we twist two adjacently contracted generators we get $T_a T_b T_a = -T_b/2N_c$ so that $(T_a T_b)^2$ is suppressed by two powers of N_c as compared to $T_a^2 T_b^2$. Then, neglecting these terms we obtain

$$L = \sum_{k=0}^{\infty} \frac{1}{(2k)!} \left(\frac{-g^2}{T^2} \right)^k \frac{\langle (A_{0,a})^{2k} \rangle}{(2N_c)^k} . \qquad (4)$$

Now, we expect $\langle (A_{0,a})^{2k} \rangle$ to scale as N_c^{2k}, in which case and for large T the limit $N_c \to \infty$ with $g^2 N_c$ fixed is finite and non-trivial, at least in the absence of radiative corrections. Finally, if we assume vacuum saturation of condensates we can reduce further the expression since

$$\langle A_0^{2k} \rangle = (2k-1)!! \langle A_0^2 \rangle^k + \text{n.v.c.}, \qquad (5)$$

where n.v.c. stands for non-vacuum–connected terms. After summing up the series we get the result

$$L = \exp \left[-\frac{g^2 \langle A_{0,a}^2 \rangle}{4 N_c T^2} \right] . \qquad (6)$$

The exponentiation means that temperature is not necessarily very high since we summed up all the series and, actually, for small T yields $L \to 0$. Of course, as one approaches the true critical temperature one expects non-analytical behaviour not encoded in the high-temperature expansion[1]. On top of eq. (6) one must take radiative corrections into account yielding, in fact, $L > 1$ [14,15] because of renormalization effects at high temperatures. This is equivalent to multiply eq. (6) by a slowly varying temperature-dependent factor [18].

The previous behaviour suggests a Gaussian ansatz for the Polyakov loop distribution, *i.e.* the probability of having a configuration with a given value of Ω. Unfortunately, not much is known about this object in QCD (see however [11]). In the Polyakov gauge, where A_0 is both static and diagonal we can write the piece of the vacuum wave functional depending on the temporal gluon fields as

$$\rho_0(A_3, A_8) = C e^{-(A_3^2 + A_8^2)/a^2}, \qquad (7)$$

where we assume that the gluon variables take any real values on the real axis. The normalization constant C and the width of the distribution a can be determined from the definition of the dimension-2 condensate

$$\langle A_3^2 + A_8^2 \rangle_0 = a^2. \qquad (8)$$

Although the previous Gaussian reproduces the result of the cumulant expansion of the Polyakov loop, it does not incorporate the known gauge invariance which corresponds to a periodicity condition on the gluon field distribution and the integration measure.

Once the Polyakov loop distribution has been determined for pure gluodynamics we set out to describe its expectation value when dynamical quarks are included. To do so, we use the PNJL model. For a constant value of the Polyakov loop, Ω, and the scalar field $S = M$ (which we identify with the constituent quark mass) the quark

[1] Nonetheless, although eq. (6) does not predict a phase transition one could still make an estimate of the critical temperature related to the point where L presents no curvature, *i.e.* the inflexion point which yields, $T_c^2 = g^2 \langle A_0^2 \rangle / 6 N_c$. On the other hand, we expect the condensate to scale with N_c and the number of Euclidean dimensions as $g^2 \langle A_\mu^2 \rangle_\mu \sim 4\pi\alpha(\mu)(N_c^2-1) D \Lambda_{\mathrm{QCD}}^2$. This shows that the large-N_c limit of L is well defined for *fixed* T. Taking $\Lambda_{\mathrm{QCD}} = 240\,\mathrm{MeV}$ and the scale $\mu = 2\,\mathrm{GeV}$ with the conversion factor $\mu = 2\pi T$ and $N_c = 3$ we get $g^2 \langle A_\mu^2 \rangle = (2.2\,\mathrm{GeV})^2$ in rough agreement with the determinations from lattice data. Then $T_c^2 \sim 4\pi\alpha(2\pi T_c)(N_c^2-1)\Lambda_{\mathrm{QCD}}^2/6N_c$ yielding the estimate $T_c = 1.1 \Lambda_{\mathrm{QCD}}$.

effective action is given by

$$\frac{T}{V}\Gamma_Q = \frac{1}{4G_S}\,\mathrm{Tr}_f(M-\hat{M}_0)^2 - 2N_f\int\frac{d^3k}{(2\pi)^3} \times\Big(N_c\epsilon_k + T\,\mathrm{tr}_c\log\Big[1+e^{-\epsilon_k/T}\Omega\Big] + \mathrm{c.c.}\Big), \quad (9)$$

where we have only retained the vacuum contribution, so there is no contribution of meson fields. V is three-dimensional volume and $\epsilon_k = +\sqrt{k^2+M^2}$ is the energy of a constituent quark with mass M. We define the Polyakov-loop–averaged action

$$e^{-\Gamma_Q(M,T)} = \int d\Omega\,\rho_0(A_3,A_8)\,e^{-\Gamma_Q(M,\Omega,T)}. \quad (10)$$

The value of M is determined by minimization of $\Gamma_Q(M,T)$ with respect to M, $\partial\Gamma_Q(M,T)/\partial M = 0$, which is the gap equation and determines M at a given temperature T, denoted as $M^* = M(T)$. This procedure allows to compute the integration in meson fields at the mean-field level. In addition, the relation between the (single flavour) chiral quark condensate, $\langle\bar{q}q\rangle$, and the constituent quark mass, reads

$$2G_S\langle\bar{q}q\rangle^* = -(M^*-m_q). \quad (11)$$

Any observable is obtained by using the temperature dependent mass M^*. For instance the expectation value of the Polyakov loop is computed as

$$\langle\Omega\rangle = \frac{\int d\Omega\,\rho_0(A_3,A_8)\,e^{-\Gamma_Q(M^*,\Omega,T)}\,\Omega}{\int d\Omega\,\rho_0(A_3,A_8)\,e^{-\Gamma_Q(M^*,\Omega,T)}}. \quad (12)$$

The integral in $d\Omega$ in the case $N_c = 3$ and in the Polyakov gauge can be computed numerically. At the mean-field level the Polyakov distribution becomes a delta function, but this picture is obviously only valid at high enough temperature, where quantum effects are negligible. At low temperatures the mean-field approximation can lead to wrong results. For example, for the expectation value of the Polyakov loop in the adjoint representation one obtains $-1/(N_c^2-1)$, instead of zero, as has been observed in lattice data [19]. The color group integration solves this problem [11]. Finally, we have taken into account here the local effects in the Polyakov loop by considering a confinement correlation volume $V_\sigma = 8\pi T^3/\sigma^3$, which follows from the simplest correlation of two Polyakov loops

$$\frac{1}{T}\int d^3x\int d\Omega\,\mathrm{tr}_c\,\Omega(\boldsymbol{x})\,\mathrm{tr}_c\,\Omega^\dagger(\boldsymbol{y}) = \frac{1}{T}\int d^3x e^{-\sigma|\boldsymbol{x}-\boldsymbol{y}|/T}. \quad (13)$$

Higher correlation functions are expected to have a smaller contribution [11].

The result for the chiral condensate and Polyakov loop expectation value is presented in fig. 1. We consider $\sqrt{\sigma} = 425\,\mathrm{MeV}$ and $m_u = m_d \equiv m_q = 5.5\,\mathrm{MeV}$. As we can see the agreement is reasonable although the quark model predicts a shift in the critical temperature of about 50 MeV. This discrepancy is compatible with the model uncertainties discussed in our previous work [11] where the explicit center symmetry breaking of the pure gluonic action was disregarded. It is also compatible with the large rescaling advocated in [12]. Basically, the shift in the critical temperature reflects our lack of knowledge on the gluon action at intermediate temperatures, in particular, the feedback of quarks on the gluons. It would be most useful to develop a theoretical framework where this feedback is taken into account, as it is done, *e.g.* in the mean-field approximation [12], while retaining the fundamentally quantum and local nature of the Polyakov loop [11].

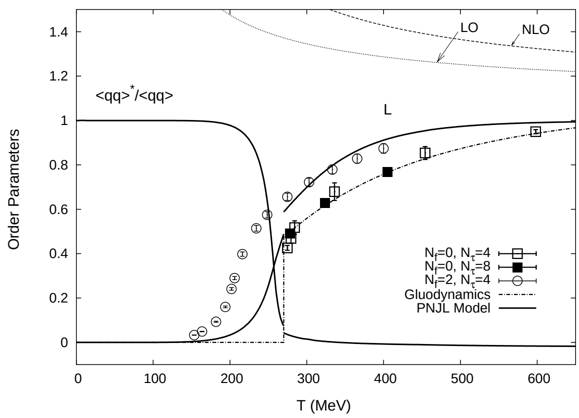

Fig. 1. Temperature dependence of chiral condensate $\langle\bar{q}q\rangle^*$ and Polyakov loop expectation value $L = \langle\mathrm{tr}_c\,\Omega\rangle/N_c$ in the 2-flavor PNJL model. We use the Gaussian distribution reproducing the Polyakov loop in the case of pure gluodynamics. Lattice data from [16] for gluodynamics ($N_f = 0$) and [17] for $N_f = 2$ full QCD. Perturbative LO and NLO results for the Polyakov loop with $N_f = 2$ are shown for comparison.

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